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coefficients. The presence of asymmetry (camber) also is accounted for. Predictions of towing configurations are presented. It is shown that small angles of attack (.15 degrees) can result in kite angles up to 60 degrees and catenary depth losses of 10 to 15 percent. Remarkably small camber (.1 percent of chord) is sufficient to cause significant kiting. Stability criteria are developed based on an analysis of the torsional buckling problems associated with the structural rigidities. The results have important implications in the design of towcable strength members and fairings.

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NOTATION

C	Chord length
C_d	Drag coefficient
C_ℓ	Lift coefficient
C_m	Moment coefficient
c_o	Camber
d	Drag per unit span
$\vec{e}_i$	Unit vector in i^{th} direction
$\delta_{s,n}$	Drag loading functions
$f = (f_s, f_k, f_n)$	Hydrodynamic force
$F = (T, F_k, F_n)$	Structural force
$K = (K_k, K_n)$	Curvature
ℓ	Lift per unit span
L	Cable length
$m = (m_s, m_k, m_n)$	Hydrodynamic moment
$M = (M_s, M_k, M_n)$	Structural moment
s	Cable arc length
(s,k,n)	Curvilinear coordinates
(x,y,z)	Cartesian coordinates
(X,Y,Z)	Cable geometry functions
V	Free stream velocity
α	Angle of attack
ε, δ	Small parameter
θ	Kite angle
(ξ, η)	Cable cross section coordinates

ρ	Fluid mass density
ϕ	Trail angle
τ	Twist

ABSTRACT

A theory for faired towcable kiting is presented. Geometric relationships and equilibrium equations are derived based on a thin elastic rod-high aspect ratio hydrofoil analytical model. The fairing section is characterized by torsional and flexural rigidities and hydrodynamic lift, drag, and moment coefficients. The presence of asymmetry (camber) also is accounted for. Predictions of towing configurations are presented. It is shown that small angles of attack (.15 degrees) can result in kite angles up to 60 degrees and catenary depth losses of 10 to 15 percent. Remarkably small camber (.1 percent of chord) is sufficient to cause significant kiting. Stability criteria are developed based on an analysis of the torsional buckling problems associated with the structural rigidities. The results have important implications in the design of towcable strength members and fairings.

ADMINISTRATIVE INFORMATION

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INTRODUCTION

The use of cable fairing in underwater cable towed systems has been successful in reducing hydrodynamic drag limitations on the speed/depth performance. Generally the fairing is a symmetric airfoil shape configured around the primary tension members in a free swivelling or integrally bonded construction. Under these circumstances, the cable may no longer act as a "taut string" for which tension and drag are the principal forces determining the catenary or towing configuration. Instead, the fairing is a lifting surface capable of developing large hydrodynamic side forces. Furthermore, the fairing structural rigidities introduce flexural and torsional loading in the towing catenary. These properties have important effects on both the steady state tracking characteristics and dynamic response of the towed system.

Faired cable is subject to a type of lateral instability, usually referred to as kiting. Kiting is the tendency of the cable to displace and remain out of the intended towing plane (usually the gravity-tow velocity plane). Previous analytical investigations, based on small perturbation/energy integral methods,[1,2] resulted in criteria for stable or kite-free

[1] Abkowitz, M.A., "The Stability of a Faired Cable of a Tethered System in its Fundamental Mode," Joseph Kaye and Co., Inc., Report 73 (Sep 1967).

[2] Hegemier, G.A., "Divergence Criteria for a Faired Towcable in a Subcavitating Flow," University of California, San Diego (Jun 1968).

towing in terms of cable physical properties and towing velocity and tension. Fairing asymmetries (camber) are not accounted for in these theories. A number of digital computer programs have been developed to compute general three dimensional towing configurations with prescribed camber distributions.[3,4] However, numerical integration is difficult when torsional and flexural loading is introduced and solutions which account for these effects are not yet available.

In this report a new treatment of towcable kiting theory is presented. A thin elastic-rod, high aspect ratio hydrofoil analytical model is used to determine the importance of both camber and destabilizing structural loading. The assumption that the cable cross section length is much smaller than the total scope and radius of curvature (thin rod) permits the use of small strain beam theory. The assumption that the spanwise variation in hydrodynamic loading is small permits the use of two-dimensional lifting surface theory. Both of these approximations are particularly appropriate to integrated towlines, which are used in high speed/deep depth towing where kiting is most severe.

The governing equations of force equilibrium and geometric compatibility are derived with particular emphasis on the role of the fairing angle of attack. By systematically ordering terms, it is deduced that the curvatures and towing catenary may be computed considering only tension and lift/drag loading. This is illustrated for several selected hydrodynamic loading functions.

The importance of the amplitude and distribution of camber is assessed and identified as a primary cause of kiting. It is shown that the chordwise locations of both the tension and hydrodynamic centers are crucial to the degree of camber induced kiting.

Further analysis considers the structural instability of a fairing in a towing catenary. Solutions in terms of known mathematical functions are derived for a selected set of cable end constraints. It is shown that under certain circumstances, torsional and flexural rigidity can substantially affect the spanwise twist distribution and resulting hydrodynamic side loading. This theory may be extended to more general cases. Conclusions are drawn which have important implications in the design of fairing and cable strength members.

GEOMETRIC RELATIONSHIPS

Consider a cable-body towed system advancing at constant speed, V, through otherwise undisturbed water. As a result of hydrodynamic, structural, and weight loading the cable forms a three-dimensional continuous arc from the body to the towing platform. The spatial configuration of the cable is defined in terms of a cartesian coordinate system (x,y,z) moving

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[3]Dillon, D.B., "The Configuration and Loading of a Torsionally Elastic Faired Cable," Hydrospace Challenger, Inc., TR-4557-0001 (Oct 1973).

[4]Wang, H.T., "A FORTRAN IV Program for the Three Dimensional Steady State Configuration of Extensible Flexible Cable Systems," Naval Ship Research and Development Center, Report 4384 (Sep 1974).

with the cable with origin at the body. The x-axis is parallel to the free stream, positive into the flow; y is directed opposite to the gravity vector; and z completes the right hand system. If s denotes arc length measured from the origin, then the geometry of the cable arc can be specified parametrically as [X(s), Y(s), Z(s)] in terms of the trail angle, $\phi(s)$, and the kite angle, $\theta(s)$, defined in Figure 1.

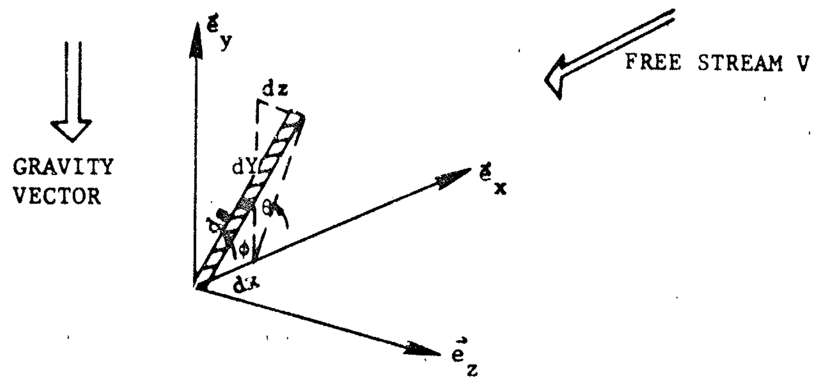

Figure 1 - Geometry of Cable Arc Element

The displacement-angle relationships are

$$\frac{dX}{ds} = \cos\phi \tag{1}$$

$$\frac{dY}{ds} = \sin\phi \cos\theta \tag{2}$$

$$\frac{dZ}{ds} = \sin\phi \sin\theta \tag{3}$$

A unit vector, $\vec{e}_s$, tangent to the arc is given by

$$\vec{e}_s = \vec{e}_x \cos\phi + \vec{e}_y \sin\phi \cos\theta + \vec{e}_z \sin\phi \sin\theta \tag{4}$$

and the curvature of the arc, $\vec{K}$, is

$$\vec{K} = \frac{d\vec{e}_s}{ds} = \vec{e}_x \left(-\sin\phi \frac{d\phi}{ds}\right) + \vec{e}_y \left(\cos\phi \cos\theta \frac{d\phi}{ds} - \sin\phi \sin\theta \frac{d\theta}{ds}\right)$$
$$+ \vec{e}_z \left(\cos\theta \sin\phi \frac{d\theta}{ds} + \cos\phi \sin\theta \frac{d\phi}{ds}\right) \tag{5}$$

It will be convenient to obtain relationships in terms of curvilinear coordinates (s,k,n), where n is normal to the cable arc in the ϕ plane and k is normal to the ϕ plane. Observe that drag forces act only in the ϕ plane while hydrodynamic lift forces act in the k direction. In the absence of kiting, the ϕ plane is the gravity-tow velocity plane everywhere along the cable. Unit vectors in the k and n directions are given by

$$\vec{e}_k = \frac{\vec{e}_x \times \vec{e}_s}{\sin\phi} = \vec{e}_y(-\sin\theta) + \vec{e}_z \cos\theta$$

$$\vec{e}_n = \vec{e}_s \times \vec{e}_k = \vec{e}_x \sin\phi + \vec{e}_y(-\cos\phi\cos\theta) + \vec{e}_z(-\cos\phi\sin\theta)$$

and the corresponding components of curvature K_k and K_n by

$$K_k = \vec{e}_k \cdot \vec{K} = \sin\phi \frac{d\theta}{ds} \tag{6}$$

$$K_n = \vec{e}_n \cdot \vec{K} = \frac{-d\phi}{ds} \tag{7}$$

The twist (change in rotation about the s-axis per unit length s) at any location along the cable is related to the kite angle and, α, the local angle of attack between the fairing chordline and the flow (ϕ plane), as illustrated in Figure 2.

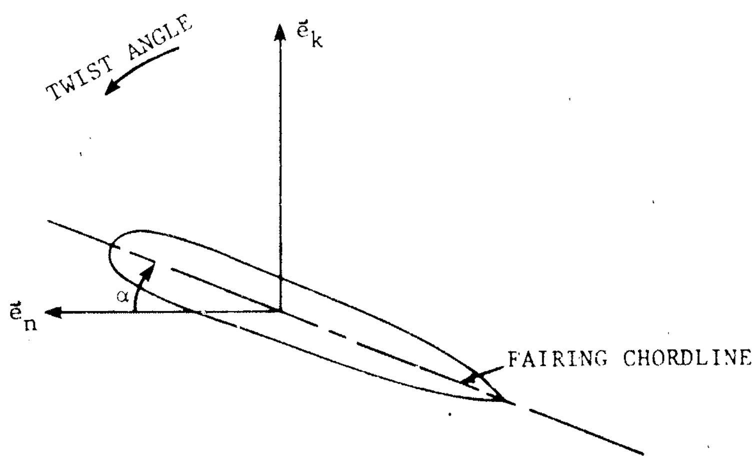

Figure 2 - Fairing Section Angle of Attack

The total twist, τ, may be written as

$$\tau = \frac{-d\alpha}{ds} + \vec{e}_n \cdot \frac{d\vec{e}_k}{ds}$$

$$= \frac{-d\alpha}{ds} + \cos\phi \frac{d\theta}{ds} \qquad (8)$$

Note that for a vertical cable ($\phi = 90°$) or a cable without kite ($\theta = 0$) the twist angle is simply $-\alpha$.

FORCE AND MOMENT EQUATIONS

The steady state loading on the cable consists of hydrostatic, hydrodynamic, and structural forces and moments. To simplify the analysis, hydrostatic loading is neglected. Thus the cable is assumed to be neutrally bouyant and to have no hydrostatic couple exerted on the fairing.

If $\vec{F}(s)$ and $\vec{M}(s)$ are the structural force and moment at any cross section along the cable, then by considering a small element, Δs, and taking the limit as $\Delta s \to 0$, the equations of static equilibrium may be written as

$$\frac{d\vec{F}}{ds} + \vec{f} = 0 \qquad (9)$$

$$\frac{d\vec{M}}{ds} + \vec{e}_s \times \vec{F} + \vec{m} = 0 \qquad (10)$$

where $\vec{f}(s) = (f_s, f_k, f_n)$ and $\vec{m}(s) = (m_s, m_k, m_n)$ are the hydrodynamic force and moment per unit length. These are the classical equations for the loading of a thin rod.[5] Assuming that $\vec{M}$, $\vec{f}$, and $\vec{m}$ can be expressed as functions of the cable angles ϕ, θ, and α, equations (9) and (10) are six ordinary differential equations for the variables $\vec{F}(s)$, $\phi(s)$, $\alpha(s)$,

$$s: \frac{dT}{ds} - F_k K_k - F_n K_n + f_s = 0 \qquad (9)(a)$$

$$k: \frac{dF_k}{ds} - F_n K_s + T K_k + f_k = 0 \qquad (9)(b)$$

$$n: \frac{dF_n}{ds} + F_k K_s + T K_n + f_n = 0 \qquad (9)(c)$$

$$s: \frac{dM_s}{ds} - M_k K_k - M_n K_n + m_s = 0 \qquad (10)(a)$$

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[5] Hildebrand, F., "Advanced Calculus for Applications," Chapter 6, Prentice-Hall, Inc., New Jersey (1962).

$$k: \frac{dM_k}{ds} - M_n K_s + M_s K_k - F_n + m_k = 0 \qquad (10)(b)$$

$$n: \frac{dM_n}{ds} + M_k K_s + M_s K_n + F_k + m_n = 0 \qquad (10)(c)$$

where $\vec{F} = (T, F_k, F_n)$, $\vec{M} = (M_s, M_k, M_n)$, and $K_s = \tau + \frac{d\alpha}{ds}$. Here, the following identities for the derivatives of unit vectors have been used:

$$\frac{d\vec{e}_s}{ds} = \vec{e}_k K_k + \vec{e}_n K_n$$

$$\frac{d\vec{e}_k}{ds} = -\vec{e}_s K_k + \vec{e}_n K_s$$

$$\frac{d\vec{e}_n}{ds} = -\vec{e}_s K_n - \vec{e}_k K_s$$

The hydrodynamic loading is approximated using high aspect ratio hydrofoil theory. Since the fairing chord length, C, is extremely small compared to the characteristic length of spanwise lift and drag variation, vortex wake induced effects on cable loading are minimal. Thus, the force and moment are functions of the local orientation to the free stream, V, and the section characteristics $C_d(\alpha)$, $C_\ell(\alpha)$, and $C_m(\alpha)$. For small angles of attack,

$$d(\alpha) \sim (C_d)_{\alpha = 0} \; \rho/_2 V^2 C \qquad (11)$$

$$\ell(\alpha) \sim \left(\frac{dC_\ell}{d\alpha}\right)_{\alpha = 0} \rho/_2 V^2 C \cdot \alpha \qquad (12)$$

$$m(\alpha) \sim \left(\frac{dC_m}{d\alpha}\right)_{\alpha = 0} \rho/_2 V^2 C^2 \cdot \alpha \qquad (13)$$

where d, ℓ, and m denote drag, lift, and moment per unit span length. The flow velocity component normal to the cable, $V\vec{e}_x \cdot \vec{e}_n = V \sin\phi$, is assumed to be the "effective velocity" in producing lift. Thus

$$\vec{f} = [-d\,\sigma_s(\phi),\ \ell(\alpha)\sin^2\phi,\ -d\,\sigma_n(\phi)] \qquad (14)$$

$$\vec{m} = [m(\alpha)\sin^2\phi,\ 0,\ 0] \qquad (15)$$

where $\sigma_s(\phi)$ and $\sigma_n(\phi)$ are hydrodynamic drag loading functions. In recent years, theoretical and experimental analyses of these functions have been conducted for a variety of fairing shapes and constructions. Two reviews of this subject are given in references 6 and 7. For purposes of analyzing towline kiting, several representative functions have been selected. These are introduced later in this report.

Since the section chord is much smaller than the radius of curvature, the cable behaves as a thin rod. Except possibly near attachment points, the strains will be small and it can be assumed that the stress-deformation relationships follow the elastic laws for the materials.[8] Thus, the structural moment, $\vec{M}$, can be expressed in terms of curvature and twist by elementary elastic beam theory. For a symmetric integrated towline the properties of the materials and cross section shape needed for the analysis are shown in Figure 3 and Table 1.

In terms of curvatures K_1 and K_2, $\vec{M}$ may be written as

$$\overline{M} = \vec{e}_s\overline{GJ}\tau + \vec{e}_1[-T \, . \, (\xi_T - \xi_s) - \overline{EI}_1K_2] + \vec{e}_2\overline{EI}_2K_1 \tag{16}$$

where s is chosen to be along the elastic axis (shear center). An equivalent form of equation (16) in (s,k,n) components, linearized in α, is

$$\vec{M} = \vec{e}_s \, \overline{GJ} \, \tau + \vec{e}_k[\alpha K_k(\overline{EI}_2 - \overline{EI}_1) - \overline{EI}_1K_n - T(\xi_T - \xi_s)] + \vec{e}_n[\overline{EI}_2K_k - \alpha K_n(\overline{EI}_2 - \overline{EI}_1) + \alpha T\xi_T] \tag{17}$$

Equations (9), (10), (14), (15), and (17) form the problem for a kiting towline. Subject to appropriate boundary conditions at the towed body (s = 0) and the towing platform (s = L), the equations can, in principle, be solved for structural loading, $\vec{F}$, and configuration ϕ, θ, and α.

KITING WITH A CONSTANT ANGLE OF ATTACK

The six equations (9) and (10) can be reduced to four by solving for the shear forces F_n and F_k in (10)(b) and (10)(c) and substituting into (9)(b) and (9)(c). This yields three force equations,

$$\frac{dT}{ds} + f_s = K_n \frac{dM_k}{ds} - K_k \frac{dM_n}{ds} - (\cot \phi)(K_k)(K_nM_n + K_kM_k) \tag{18}$$

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[6]Casarella, M. J. & Parsons, M., "Cable Systems Under Hydrodynamic Loading," MTS Journal Vol. 4, No. 4 (Jul-Aug 1970).

[7]Folb, R., "Experimental Determination of Hydrodynamic Loading for Ten Cable Fairing Models," SPD R&D Report 4610 (in progress)

[8]Love, A.E.H., "A Treatise in the Mathematical Theory of Elasticity," Chapter 18, Dover, New York (1944).

TABLE 1

Definitions of Cable Structural Coefficients

Coefficient	Symbol	Formula
Torsional Rigidity	$\overline{GJ}$	$\int_{AREA} (G)[(\xi - \xi_s)^2 + \eta^2]dA$
Chordwise Flexural Rigidity	$\overline{EI}_1$	$\int_{AREA} (E)(\xi - \xi_s)^2 dA$
Lateral Flexural Rigidity	$\overline{EI}_2$	$\int_{AREA} (E)(\eta^2)dA$
Tension Center	$(0, \xi_T)$	$\frac{\int_{AREA} (E)(\xi)dA}{\int_{AREA} (E)dA}$
Shear Modulus	G	-------
Elastic Modulus	E	-------

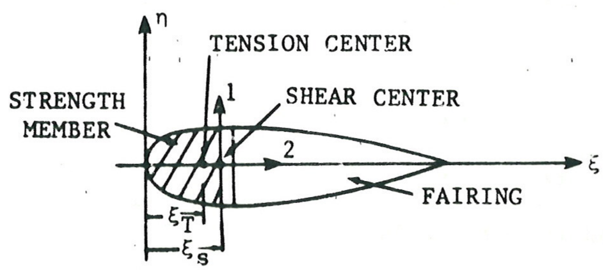

Figure 3 - Fairing Cross Section

$$TK_k + f_k = (\cot\phi)(K_k)\left(\frac{dM_k}{ds} - K_k M_n \cot\phi + M_s K_k\right)$$

$$+ \frac{d}{ds}\left(\frac{dM_n}{ds} + M_s K_n + M_k K_k \cot\phi\right) \tag{19}$$

$$TK_n + f_n = (\cot\phi)(K_k)\left(\frac{dM_n}{ds} + M_s K_n + M_k K_k \cot\phi\right)$$

$$- \frac{d}{ds}\left(\frac{dM_k}{ds} - K_k M_n \cot\phi + K_k M_s\right) \tag{20}$$

and the twist moment equation

$$\frac{dM_s}{ds} - M_k K_k - M_n K_n + m_s = 0 \tag{21}$$

To compare magnitudes of the various terms, the tension, T, is scaled by tension at the body, T_o. An appropriate length scale for s is T_o/d. It will be seen that this length is a measure of the radius of curvature at the towed body. The non-dimensional force equations (18), (19), and (20) are

$$\frac{dT}{ds} + \sigma_s = O(\varepsilon) \tag{22}$$

$$T \sin\phi \frac{d\theta}{ds} + \frac{\ell \sin^2\phi}{d} = 0\ (\varepsilon) \tag{23}$$

$$T\frac{d\phi}{ds} + \sigma_n = O(\varepsilon) \tag{24}$$

where the expression $O(\varepsilon)$ denotes terms of order $\frac{\overline{EI}_1 d^2}{T_o^3}$, $\frac{\overline{EI}_2 d^2}{T_o^3}$ and $\frac{\overline{GJ} d^2}{T_o^3}$. Generally, for those circumstances in which kiting occurs,

these parameters are several orders of magnitude less than the terms on the left hand sides. Thus, over most of the cable span, the $O(\varepsilon)$ terms can be ignored. This amounts to assuming that structural rigidity is ignored or that the cable is flexible. Except for small regions near the cable ends, this is a valid approximation in determining solutions for ϕ, θ, and T.

Equations (21) and (23) (with right hand side set to zero) are identical to the two-dimensional (planar) towing equations for a flexible cable, provided that the drag loading function assumptions (equations (11) and (14)) are valid. Once T(s) and $\phi(s)$ are solved for, equation (23) relates the kite angle θ to the section lift/drag ratio, ℓ/d, in the form

$$\frac{d\theta}{ds} = \frac{\ell}{d}\,\frac{\sin\phi}{T} \tag{25}$$

Note that for $\ell/d \sim O(1)$, substantial kiting is possible, as would be expected from physical reasoning. At the same time, $\ell/d \sim O(1)$ implies very small angles of attack, viz.:

$$\alpha \sim \frac{\ell}{d}\,\frac{C_d}{\left(\frac{dC_\ell}{d\alpha}\right)_{\alpha=0}} \sim (1)\,\frac{.02}{2\pi} \sim .003_{\text{RADIANS}} \tag{26}$$

In general $\alpha(s)$ must be solved from the twist moment equation (21). Under those circumstances when $\alpha(s)$ is a constant (these will be determined later) so is ℓ/d and equation (25) is easily integrated. Solutions for ϕ, T, and θ, corresponding to four selected drag loading functions, are summarized in Table 2. For simplicity, the boundary conditions at the body have been selected to be $\phi(o) = 90$ degrees (body drag/lift ratio is zero) and $\theta(o) = 0$ (body exerts no lateral force). The towing catenaries corresponding to the case $\alpha \equiv o$ are shown in Figure 4. The kite angle is shown as a function of scope in Figure 5.

This angle is weakly dependent on the particular form of loading function. Also, for reasonable scopes, the variation in kite from the extreme case of a vertical trail is small. For a scope $\frac{Sd}{T_o} = 1.0$, the lateral displacement as a function of cable lift-to-drag ratio is shown in Figure 6. The depth loss due to kiting is shown in Figure 7. This is normally less than 10 percent, even at kite angles up to 45 degrees. In summary, the effect of a small constant angle of attack is a substantial lateral body displacement, large kite angles in the upper part of the towline, and a small loss in depth. No change in trail occurs if the increase in drag due to angle of attack is neglected.

EFFECT OF CAMBER

Fairings are usually designed as symmetric hydrofoils sections. The presence of asymmetry (or camber) in the fairing cross section shape due, for example, to manufacturing limitations or deformation under use is recognized as a cause of kiting. If torsional and flexural stiffness effects on the towline twist are neglected, a simple physical interpretation of the mechanism of camber induced kiting and formulas for predicting the kiting can be derived.

Table 2 - Loading and Configuration Functions for Kiting Towline With Constant Angle of Attack

ITEM		1	2	3	4
TANGENTIAL LOADING	σ_s/d	$\sin\phi\cos\phi$	0	$\cos\phi$	0
NORMAL LOADING	σ_n/d	$\sin^2\phi$	$\sin^2\phi$	$\sin\phi$	$\sin\phi$
TENSION	T/T_o	$\csc\phi$	1	$\csc\phi$	1
SCOPE	sd/T_o	$\frac{1}{2}(\ln[\cot\phi/2] + \cot\phi\csc\phi)$	$\cot\phi$	$\cot\phi$	$\ln[\cot\phi/2]$
TRAIL	xd/T_o	$\frac{1}{2}\cot^2\phi$	$\csc\phi - 1$	$\csc\phi - 1$	$\ln[\csc\phi]$
DEPTH $\vert_{\alpha=0}$	Yd/T_o	$\cot\phi$	$\ln[\cot\phi/2]$	$\ln[\cot\phi/2]$	$\pi/2 - \phi$
KITE ANGLE	$\theta d/\ell$	$\ln[\tan\phi/2]$	$\ln[\tan\phi/2]$	$\phi - \pi/2$	$\phi - \pi/2$
DEPTH	Yd/T_o	$\frac{1}{1+\left(\frac{\ell}{d}\right)^2}\cot\phi\cos\theta - \frac{\ell/d}{1+\left(\frac{\ell}{d}\right)^2}\sin\theta\csc\phi$	$-(d/\ell)\sin\theta$	$\int_0^{\pi/2-\phi}\frac{\cos\left(\frac{\ell}{d}r\right)}{\cos r}dr$	$-\left(\frac{d}{\ell}\right)\sin\theta$
SIDE TRAIL	Zd/T_o	$\frac{1}{1+\left(\frac{\ell}{d}\right)^2}\cot\phi\sin\theta + \frac{\ell/d}{1+\left(\frac{\ell}{d}\right)^2}(\cos\theta\cos\phi-1)$	$(d/\ell)(\cos\theta - 1)$	$-\int_0^{\pi/2-\phi}\frac{\sin\frac{\ell}{d}r}{\cos r}dr$	$(d/\ell)(\cos\theta - 1)$

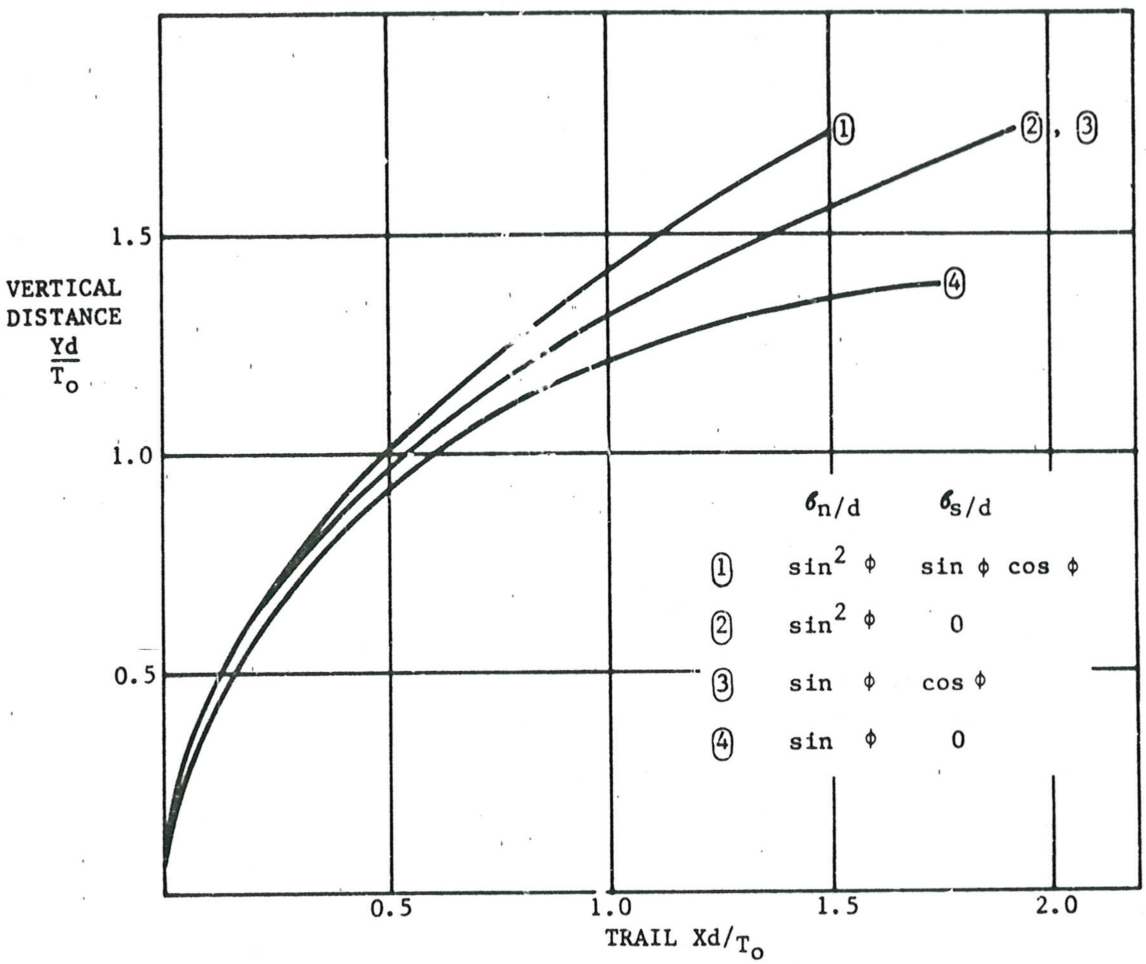

Figure 4 - Nondimensional Catenaries for Four Loading Functions

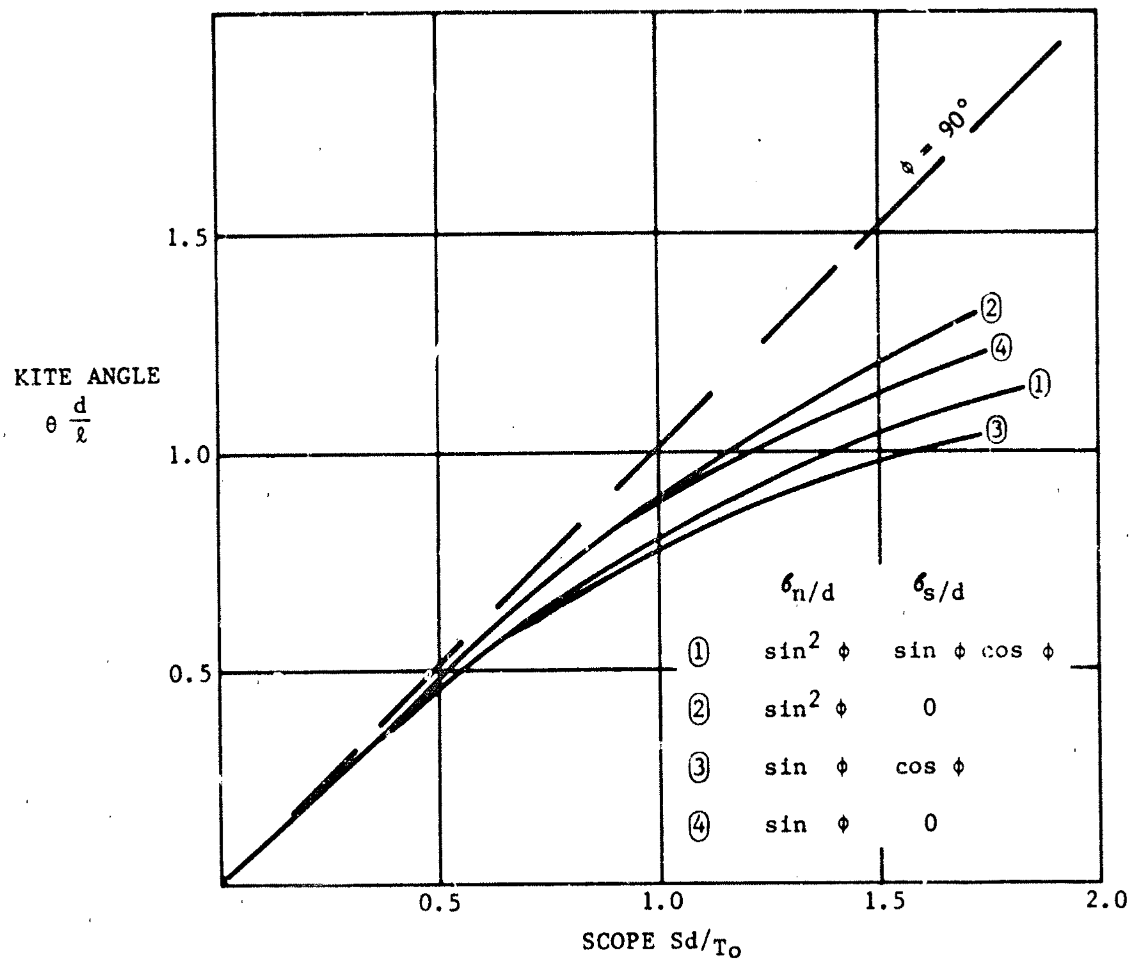

Figure 5 - Kite Angle as a Function of Scope

Consider the twist moments and kiting forces on a section of a cambered towline initially at zero angle of attack. The fairing will rotate until an equilibrium of moments is reached at some angle α_e, such that $m_s(\alpha_e) = 0$. (The moment in this case is referred to the effective center of tension.) In general, however, the kiting force $f_k(\alpha_e)$ is not zero. Thus, the towline will displace laterally until the tension force (TK_k) balances $f_k(\alpha_e)$ (equation (23)). This is shown schematically in Figure 8, where, for simplicity the lift due to camber ℓ_c, and angle of attack ℓ_α, are shown as separate, additive effects.

For moment equilibrium,

$$\ell_\alpha(\xi_\alpha - \xi_T) - \ell_c(\xi_c - \xi_T) = 0 \tag{27}$$

or

$$\ell_\alpha = \ell_c \frac{\xi_c - \xi_T}{\xi_\alpha - \xi_T}$$

where ξ_α, ξ_c, and ξ_T are the chordwise positions of the hydrodynamic and tension forces measured from the leading edge. The hydrodynamic kiting force, f_k, is

$$f_k = \ell_c - \ell_\alpha \tag{28}$$

$$= \ell_c \left(1 - \frac{\xi_c - \xi_T}{\xi_\alpha - \xi_T}\right)$$

In terms of the camber lift coefficient C_c defined as

$$C_c \equiv \ell_c/\rho/_2 V^2 C \tag{29}$$

the lift-to-drag ratio may be written as

$$\ell/d = f_k/d = C_c/C_d \left(1 - \frac{\xi_c - \xi_T}{\xi_\alpha - \xi_T}\right) \tag{30}$$

and the angle of attack α_e

$$\alpha_e = \frac{C_c}{\left(\frac{dC_\ell}{d\alpha}\right)_{\alpha = 0}} \cdot \frac{\xi_c - \xi_T}{\xi_\alpha - \xi_T} \tag{31}$$

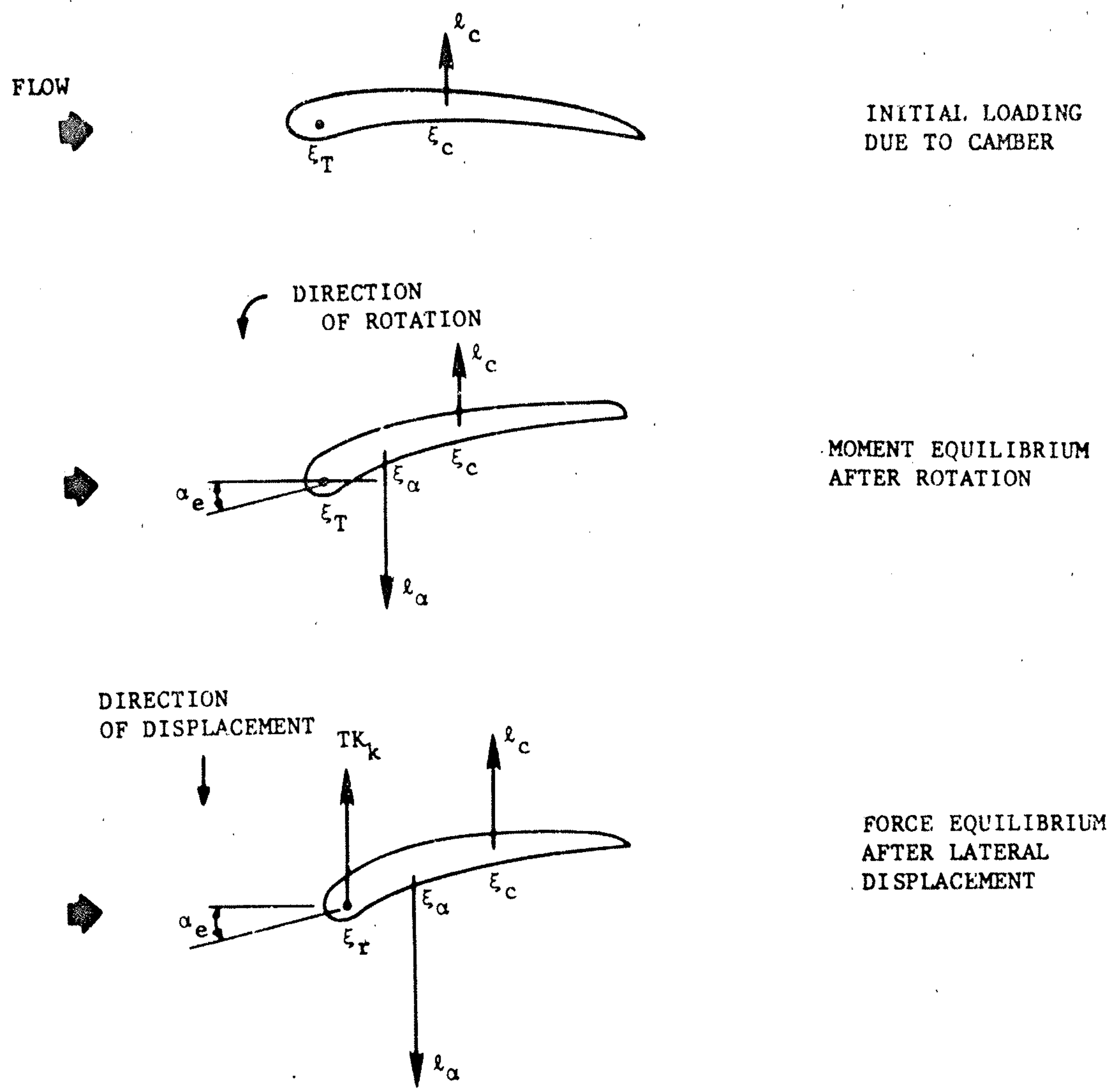

Figure 8 - Camber Induced Kiting

In the linear approximation, C_c is proportional to the camber/chord ratio, $\frac{C_o}{C}$. Thus, for a prescribed camber distribution and drag coefficient,

$$\frac{\ell}{d} \propto \left(\frac{C_o}{C}\right)\left(1 - \frac{\xi_c - \xi_T}{\xi_\alpha - \xi_T}\right) \tag{32}$$

This relationship is graphically presented in Figure 9 for a parabolic camber profile. Extermely small camber ratios, constant along the span, result in sufficient angles of attack and corresponding lift/drag ratios to cause significant kiting. The sensitivity to position of center of tension and center of pressure also is apparent. The center of pressure is primarily a function of the shape and construction of the fairing. The center of tension is located by the choice of shape, material and construction of the cable strength member.

It should be noted that a spanwise sinusiodal distribution of camber, with sufficiently small wavelength, will substantially reduce the degree of kiting. For example, a vertical cable (ϕ = 90°) of length L with angle of attack $\alpha = \alpha_o \sin n\pi S/L$ (n = integer) has a lateral displacement Z(o), due to kiting of

$$\frac{Z(o)}{L} = \left(-\frac{dC_\ell}{d\alpha}\right)_{\alpha = 0} \cdot \frac{\rho/_2 V^2 CL}{T_o} \cdot \frac{\alpha_o}{n\pi} \tag{33}$$

EFFECT OF FLEXURAL RIGIDITY

In an integrated towline, the strength member is usually shaped such that bending in the towing catenary introduces destabilizing torsional moments. This is a consequence of the fairing streamlined shape and the desire to fill as much of the area as possible with tension carrying fibers. It is of interest to determine under what circumstances the towline is subject to torsional buckling instability. That is, the towline, if subjected to small disturbances, will assume divergent angles of attack and subsequent kiting.

If torsional rigidity effects are neglected ($\overline{GJ} \equiv 0$), the twist equation with s chosen as the elastic axis reduces to

$$-M_k K_k - M_n K_n + m_s \geq 0; \quad \alpha > 0 \tag{34}$$

Since each cable section now acts independently in twist, the requirement for stability is a positive net moment for positive α (a restoring moment). Substituting the $\vec{M}$ components from Equation (17) and combining terms gives

$$T(\xi_T - \xi_s)(K_k - \alpha K_n) + (\overline{EI}_2 - \overline{EI}_1)(\alpha K_n^2 - \alpha K_k^2 - K_n K_k) + m_s \geq 0 \tag{35}$$

This can be written, using equations (19) and (20) as

$$T(\xi_T - \xi_s)K_n\left(\frac{f_k}{f_n} - \alpha\right) + (\overline{EI}_2 - \overline{EI}_1)(K_n^2)\left[\alpha - \alpha\left(\frac{f_k}{f_n}\right)^2 - \frac{f_k}{f_n}\right] + m_s \geq 0 \tag{36}$$

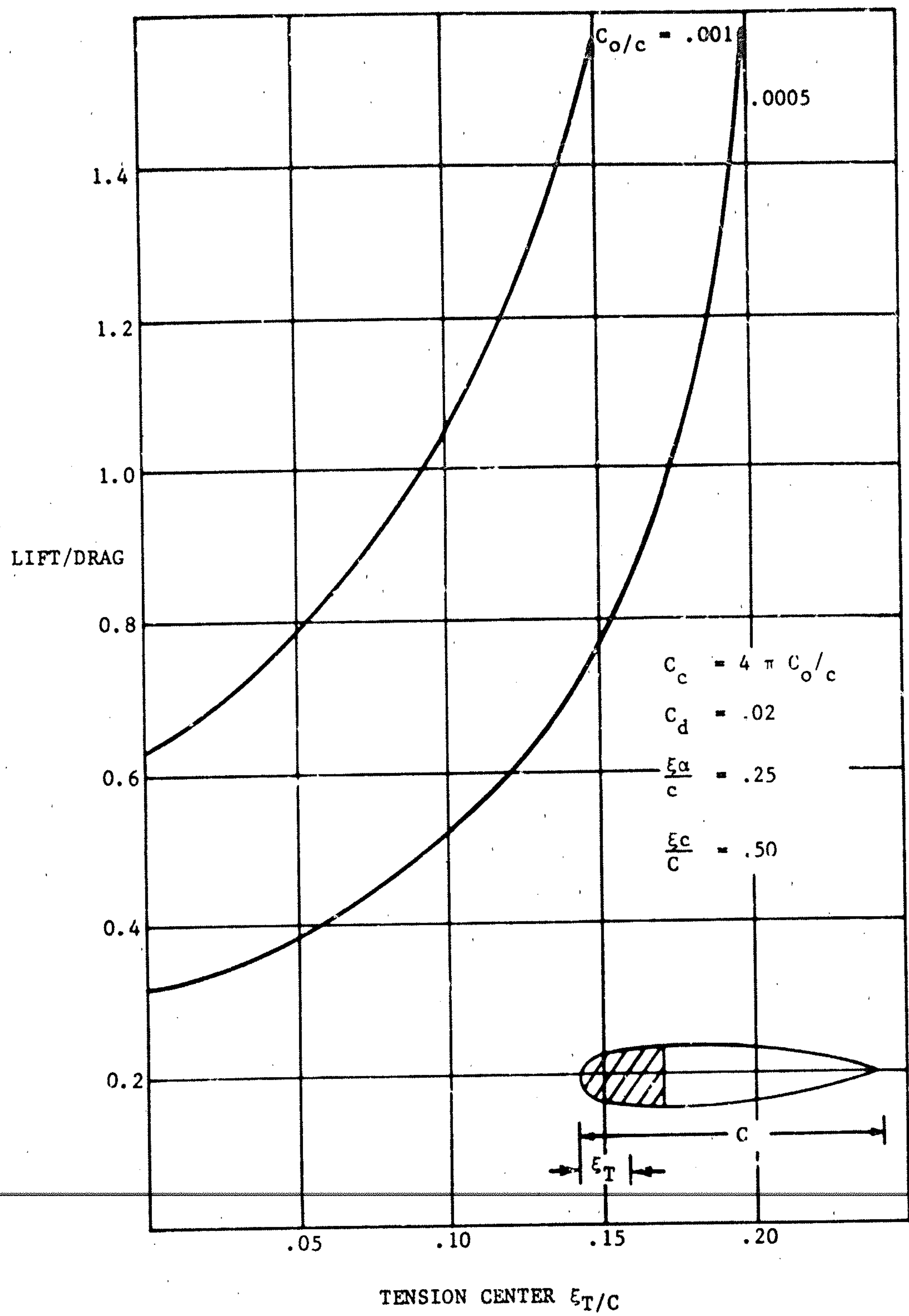

Figure 9 - Lift/Drag of a Kiting Faired Cable With Parabolic Camber

In the stability analysis, we are concerned with cases of $f_k/f_n < 1$ and since $\alpha << f_k/f_n$, equation (36) simplifies to

$$T(\xi_T - \xi_s) K_n \cdot \frac{f_k}{f_n} + (\overline{EI}_2 - \overline{EI}_1)(K_n^2)\left(-\frac{f_k}{f_n}\right) + m_s \geq 0 \tag{37}$$

or

$$-(\xi_T - \xi_s) + \frac{\overline{EI}_2 - \overline{EI}_1}{TR} + \frac{m_s}{f_k} \geq 0 \tag{38}$$

where R is the local radius of curvature in the ϕ plane $(-\frac{1}{K_n})$. Using equations (14) and (15), this may be written as

$$-(\xi_T - \xi_s) + \frac{\overline{EI}_2 - \overline{EI}_1}{TR} + \frac{m(\alpha)}{\ell(\alpha)} \geq 0 \tag{39}$$

If the approximation

$$m(\alpha) \approx \ell(\alpha) \cdot (\xi_\alpha - \varepsilon_s) \tag{40}$$

is used, where ξ_α is the chordwise position of the center of lift, then the requirement for twist stability becomes

$$\xi_\alpha - \xi_T \geq \frac{\overline{EI}_1 - \overline{EI}_2}{TR} \tag{41}$$

Since the structural destabilizing moment and the hydrodynamic restoring moment are both nearly linearly dependent on lift coefficient, it does not appear in equation (41). The speed of tow enters only implicity in determining the tension and radius of curvature. It may be concluded from equation (41) that flexural stiffness acts to reduce the effective hydrodynamic restoring moment arm $(\xi_\alpha - \xi_T)$. As an example of application of equation (41), consider the integrated towline developed by the Boeing Company in conjunction with the Naval Undersea Center.[9] The characteristics are:

$\overline{EI}_1$	= 25,420 lb in^2	
$\overline{EI}_2$	= 17,340 lb in^2	
T	= 10,000 lb	40 knots tow speed
R	= 930 ft	40 knots tow speed

[9]Calkins, D.E., "Hydrofoil High-Speed Towed System: Trial Evaluation, Part III," NUC TP 241 (Aug 1972).

$$\xi_\alpha - \xi_T \qquad = 0.269 \text{ inch}$$

Inserting these values into equation (41) yields,

$$\xi_\alpha - \xi_T \geq 0.0000724 \text{ inch}$$

which is obviously satisfied.

EFFECT OF TORSIONAL RIGIDITY

The effect of torsional rigidity, $\overline{GJ}$, can be determined by solving the complete twist equation (21). Substituting the $\hat{M}$ components from equation (17) and m_s from equation (15) yields

$$\overline{GJ}\,\frac{d\tau}{ds} + (\overline{EI}_2 - \overline{EI}_1)(\alpha K_n^{\,2} - \alpha K_k^{\,2} - K_n K_k) + T(\xi_T - \xi_s)(K_k - \alpha K_n) + m(\alpha)\sin^2\phi = 0 \tag{42}$$

Closed form solutions to equation (42) for general functions T(s) and ϕ(s) are not possible. A simpler but useful problem can be formulated by assuming that the basic trail catenary is nearly vertical and with a radius of curvature large compared to total scope, L. Thus if $dL/T_o = \delta$ is considered a small parameter, then the following approximations are valid:

$$\begin{aligned} T/T_o &= 1 + O(\delta^2) \\ L \cdot K_n &= \frac{d\phi}{ds} = -\delta + O(\delta^2) \\ L \cdot K_k &= -\delta(\ell/d) + O(\delta^2) \\ L \cdot \tau &= \frac{-d\alpha}{d\mathit{s}} - \delta^2 \mathit{s}\,\frac{\ell}{d} + O(\delta^3) \end{aligned} \tag{43}$$

where $\mathit{s} = s/L$. Inserting these expressions into equation (42) regarding $\ell/d \sim O(1)$ and retaining terms of order α and δ^2 gives

$$\frac{d\alpha}{d\mathit{s}} - \frac{d\alpha}{d\mathit{s}} + \delta^2 \mathit{s}\,\frac{\ell(\alpha)}{d} + \left(\frac{\frac{T_o^{\,2}}{d}(\xi_T - \xi_s) + \overline{EI}_2 - \overline{EI}_1}{\overline{GJ}}\right)\delta^2\,\frac{\ell(\alpha)}{d} + \frac{m(\alpha)}{\overline{GJ}}\,L^2 = 0 \tag{44}$$

If the approximation of equation (40) is used for m(α) and equation (12) for $\ell(\alpha)$, then in terms of the variable s*,

$$s^* = \mathit{s}\,\delta\sqrt{\frac{\left(\frac{dC_\ell}{d\alpha}\right)}{C_d}}\;\alpha = 0 \tag{45}$$

one obtains, after some manipulation,

$$\frac{d^2\alpha}{ds*^2} + s* \frac{d\alpha}{ds*} + Q\,\alpha = 0 \tag{46}$$

where the constant Q is given by

$$Q = \frac{\overline{EI}_1 - \overline{EI}_2 + \overline{GJ} - \frac{To^2}{d}\,(\xi_\alpha - \xi_T)}{\overline{GJ}} \tag{47}$$

The general solution of equation (46) may be written as

$$\alpha(s*;\ Q) = e^{-\frac{1}{2}s*^2} \Big\{ C_1\, m\,(\tfrac{1}{2} - \tfrac{Q}{2}, \tfrac{1}{2}, \tfrac{1}{2}\, s*^2) + C_2\, s*\, m\,(1 - \tfrac{Q}{2}, \tfrac{3}{2}, \tfrac{1}{2}\, s*^2) \Big\} \tag{48}$$

where $m(a,b,x,)$ is the confluent hypergeometric function. If the fairing is free swivelling at its terminations, $[\tau(o) = \tau(L) = 0]$ then nontrivial solutions are possible only if

$$m\,(\frac{3}{2} - \frac{Q}{2}, \frac{3}{2}, \frac{1}{2}\, s_L^{*2}) = 0 \tag{49}$$

where $s_L^* = s*$ at $s = L$. The smallest value of s_L^* for which equation (49) is satisfied may be found by using Abramowitz[10] formula for x_o, the first positive zero of $m\,(a,b,x)$,

$$x_o \approx \frac{\pi^2 \left(\frac{1}{4} + \frac{b}{2}\right)^2}{2b - 4a}$$

Applying this to Equation (49) yields the stability requirement

$$\frac{1}{2}\, s*^2 \leq \frac{\pi^2}{-3 + 2Q} \tag{50}$$

or, equivalently,

$$\xi_\alpha - \xi_T \geq -\frac{\pi^2\, \overline{GJ}}{\left(\frac{d\ell}{d\alpha}\right) L^2} + \frac{\overline{EI}_1 - \overline{EI}_2 - \frac{1}{2}\overline{GJ}}{T_o\, R_o} \tag{51}$$

where $R_o = T_o/d$ is the radius of curvature.

[10] Abramowitz, M. and I. Stegun, "Handbook of Mathematical Functions," Chapter 13, U.S. Government Printing Office, Washington, D.C., 1965

If the cable is free swivelling at $s = L$ ($\tau(L) = 0$), but built in at $s = 0$ $(\vec{e}_z \cdot \vec{e}_2 = 0)_{s=0}$, then in a similar manner one obtains the relation of equation (51) except for a factor of 1/4 on the first term of the right-hand side. These results may be compared to the extreme case of a vertical cable ($dL/T_o \rightarrow 0$) for which the twist equation reduces to

$$\frac{d^2\alpha}{ds^2} - \lambda\,\alpha = 0; \quad \lambda = \frac{\left(\frac{d\ell}{d\alpha}\right)_{\alpha=0} (\xi_\alpha - \xi_T)}{\overline{GJ}} \tag{52}$$

If $\lambda>0$, stable solutions are assured. If $\lambda<0$, then the general solution is

$$\alpha = C_1 \sin\sqrt{-\lambda}\, s + C_2 \cos\sqrt{-\lambda}\, s \tag{53}$$

and for which stability is assured if

$$\begin{aligned} -\lambda &< \frac{\pi^2}{L^2} \qquad \frac{d\alpha}{ds} = 0 \quad \text{at } s = 0,\ L \\ -\lambda &< \frac{1}{4}\,\frac{\pi^2}{L^2} \qquad \frac{d\alpha}{ds} = 0 \quad s = L \\ & \qquad\qquad\qquad \alpha = 0 \quad s = 0 \end{aligned} \tag{54}$$

These are equivalent to the solution given by equation (51) for $\frac{dL}{To} \rightarrow 0$. Thus, the effect of $\overline{GJ}$ is seen to be stabilizing in two ways. First, there is an end effect (usually very small for realistic fairing lengths) dependent on the type of end fixity. Second, there is a stabilizing term associated with the curvature in the same manner as the flexural rigidities.

It is interesting to note that in the case of a straight line catenary, if $\lambda>0$, the solution with $\alpha(0) = \alpha_o$ and $d\alpha/ds = 0$ at $S = L$ is

$$\alpha = \alpha_o \frac{\cosh\sqrt{\lambda}\,(L-s)}{\cosh\sqrt{\lambda}\,L}$$

and as $\sqrt{\lambda}\,L \rightarrow \infty$, $\frac{\alpha}{\alpha_o} \rightarrow e^{-\sqrt{\lambda}\,s}$. Thus, the effect of towed body torque (or, for that matter, a torsional disturbance anywhere along the fairing) exponentially decays from the point of application.

CONCLUSIONS

A theory for the hydrodynamic and structural mechanisms leading to faired towline kiting is presented. Stability criteria and relationships for predicting towing performance are developed in terms of the fairing section geometric properties, structural characteristics, and hydrodynamic coefficients. Specifically, it is found that:

1) A small angle of attack, constant along the faired towline is sufficient to cause catastrophic kiting. For a lift/drag ratio of unity ($\alpha \approx 0.15$ degree), kite angles up to 60 degrees can result (Figure 5), with a corresponding depth loss of 10 to 15 percent (Figure 7) and body lateral displacement of 40 percent of scope (Figure 6).

2) Remarkably small fairing section asymmetries (0.1 percent of chord), if constant spanwise, result in sufficient angle of attack for severe kiting. The degree of this camber induced kiting is critically dependent on the chordwise locations of hydrodynamic and tension loading.

3) Flexural rigidity, in combination with the towing catenary curvature can result in a destabilizing twist moment. A criteria for the required hydrodynamic moment arm to overcome this effect is given in equation (41).

4) Torsional rigidity acts to stabilize the fairing in twist, depending on end constraints and catenary curvature as shown in equation (51).

The foregoing analysis will provide criteria for designing improved faired towline strength members and fairings while minimizing the risk of kiting. The theory also provides a general framework which could be extended to investigate, for example, dynamic stability.

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